Solved Question Paper Questions graph theory

Unit 1

1.c)State and prove the handshaking theorem. [June 2017, 5 marks]

Handshaking problem :

If G is a (p,q) graph with $V(G)=\{V_1....V_p\}$ and $d_i=d_G(V_i)$, $1 \le i \le p$, then

$$2q = \sum_{i=1}^{p} d_i$$

Proof: Consider the set $S = \{(x, e): x \in V(G), e \in E(G), x \text{ is an endpoint of } e\}$. Choose a vertex $v_i \in V$. This can be done in p ways. Now, since $d_i = d(v_i)$, there are precisely d_i edges incident with this vertex v_i. These edges give d_i elements of the set S. Adding over all the vertices of G, we get

$$|S| = \sum_{i=1}^{p} d_i . \qquad (1)$$

Now choose an edge e in E (G). This can be done in q ways. This edge has precisely two endpoints, and they give two elements of S. Summing over every edge $e \in E(G)$, we get

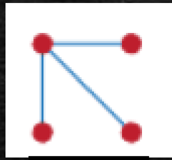
$$|S| = 2q \qquad (2)$$

This is because every edge is counted twice, once for each vertex it contains. Equating (1) and (2) we get the required result.

1.d)Define the following symbols : i) δ (G)[June 2017,1 mark]

Minimum vertex degree of a graph is δ (G). It is min{$d_G(x)$:x∈ V(G)} .

δ (G) is a non-negative integer.

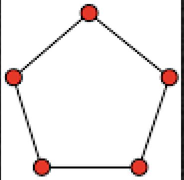


G

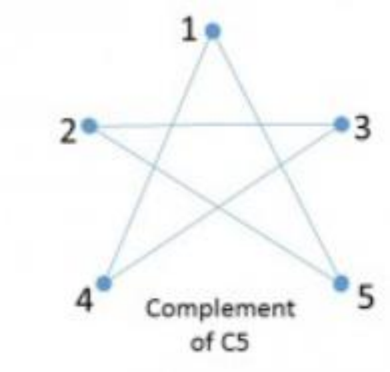
δ (G)=1

2.a)What is meant by complement of a graph ? Find the complement of the C5 graph (i.e. C5). [June 2017, 3 marks]

Complement of a graph : Let graph G=(V,E) be a (p,q) graph. Complement of the graph is a graph V($\overline{G}$) = V(G) and E($\overline{G}$)= {xy : xy ∉ E (G), x, y ∈V (G)}.

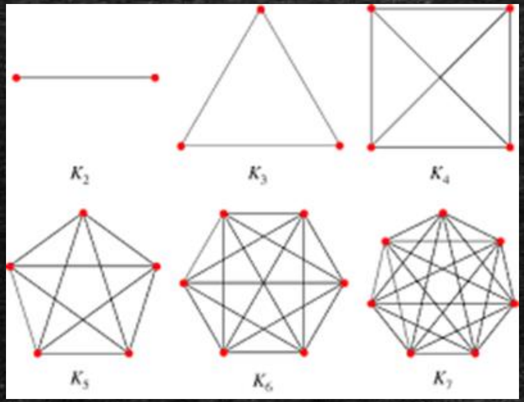


C_5



2.b) What is a complete graph ? [June 2017, 2 marks]

Complete graph : Graph in which any two vertices are adjacent, i.e. each vertex is joined to every other vertex by a vertex. A complete graph on n vertices is represented by K_n.



5.c)Define isomorphism. Determine whether the following pair of graphs are isomorphic : [June 2017, 3 marks]

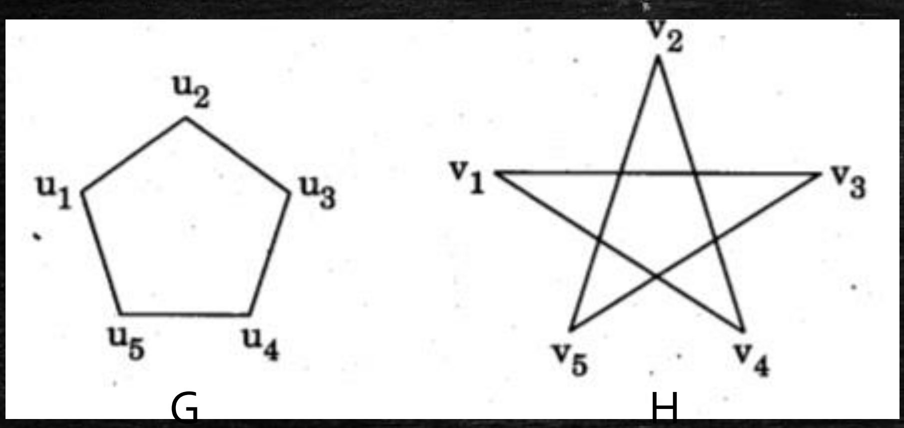
Let G=(V(G),E(G)) and H=(V(H),E(H)) be two graphs. Let us map a function f: V(G)-> V(H).

Then two graphs are said to be isomorphic, if

i) F is one-one and onto, and

ii) xy ∈ E(G) if and only if f(x) f(y) ∈ E(H)

If not they are called non-isomorphic graphs.



To check for isomorphism check the following :

1. Number of vertices

Number of vertices in G=5

Number of vertices in H=5

2. Number of edges

Number of edges in G=5

Number of edges in H=5

3. Degree sequence

Degree sequence of G : <2,2,2,2,2>

Degree of sequence of H : <2,2,2,2,2>

The above shows that degree sequence of two graphs is the same.

f(u1)=v1, f(u2)=v2, f(u3)=v3, f(u4)=v4, f(u5)=v5

From the above checks, we can conclude that the two graphs are isomorphic.

3.c)What do you mean by isomorphic graphs ?[June 2016, 2 marks]

Let G=(V(G),E(G)) and H=(V(H),E(H)) be two graphs. Let us map a function f: V(G)-> V(H).

Then two graphs are said to be isomorphic, if

i) F is one-one and onto, and

ii) xy ∈ E(G) if and only if f(x) f(y) ∈ E(H)

If not they are called non-isomorphic graphs.

To check if two graphs check for these conditions :

1. Count the number of vertices – must be equal
2. Count the number of edges – must be equal
3. Degree sequence – must be same
4. Number of cycles – must be same
5. Max degree vertex and min degree vertex
6. Peculiarity of adjacent vertices

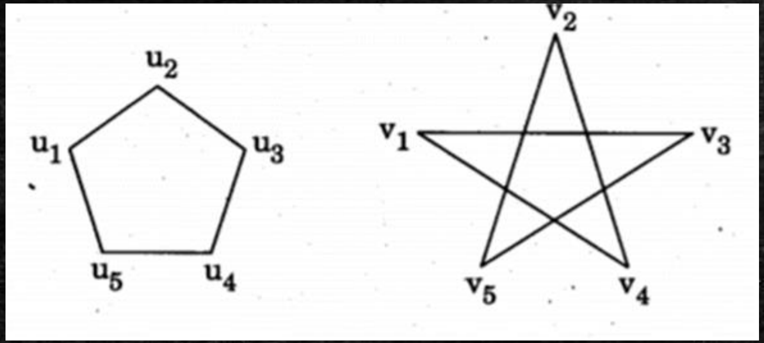
To check for isomorphism check the following :

1. Number of vertices

Number of vertices in G=5

Number of vertices in H=5

Consider the two graphs:



2. Number of edges

Number of edges in G=5

Number of edges in H=5

3. Degree sequence

Degree sequence of G : <2,2,2,2,2>

Degree of sequence of H : <2,2,2,2,2>

The above shows that degree sequence of two graphs is the same.

From the above checks, we can conclude that the two graphs are isomorphic.

4.a) State Handshaking Theorem.[June 2016,3 marks]

If G is a (p,q) graph with $V(G)=\{V_1....V_p\}$ and $d_i=d_G(V_i)$, $1\le i\le p$, then $$2q = \sum_{i=1}^{p} d_i$$

Proof: Consider the set $S = \{(x, e): x \in V(G), e \in E(G), x \text{ is an endpoint of } e\}$. Choose a vertex $v_i \in V$. This can be done in p ways. Now, since $d_i = d(v_i)$, there are precisely d_i edges incident with this vertex v_i. These edges give d_i elements of the set S. Adding over all the vertices of G, we get

$$|S| = \sum_{i=1}^{p} d_i \,. \qquad (1)$$

Now choose an edge e in E (G). This can be done in q ways. This edge has precisely two endpoints, and they give two elements of S. Summing over every edge $e \in E(G)$, we get

$$|S| = 2q \qquad (2)$$

This is because every edge is counted twice, once for each vertex it contains. Equating (1) and (2) we get the required result.

4.b) A non-directed graph G has 8 edges. Find the number of vertices, if the degree of each vertex in G is 2. [June 2016, 3 marks]

According to the formula,

$$2q = \sum_{i=1}^{p} d_i$$

q=8

Sum of degree of all vertices < = 2 * no. of edges . [According to Handshaking theorem]

Let n be number of vertices in graph.
==> 2*n = 2*8
==> 2n=16

==> n=8

1.b)Prove that the complement of $\overline{G}$ is G.[December 2016,5 marks]

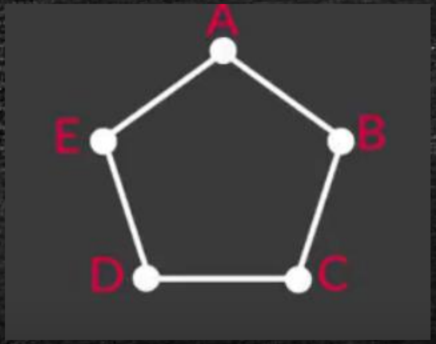
Let graph G=(V,E) be a (p,q) graph. Complement of the graph is a graph V($\overline{G}$) = V(G) and E($\overline{G}$)= {xy : xy ∉ E (G), x, y ∈V (G)}.

From the above definition, we can say that complement of a graph $\overline{G}$ has,

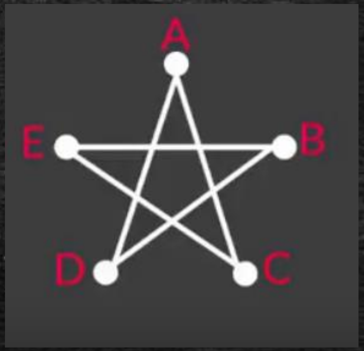
V( )=V(G) and E($\overline{G}$)= {xy : xy ∉ E (G), x, y ∈V (G)}.
Complement of $\overline{G}$ is G
(V($\overline{G}$))'=V(G) and (E$\overline{G}$))'={xy : xy ∉ E ($\overline{G}$), x, y ∈V (G)}=E(G)

Hence proved.

Example :



G



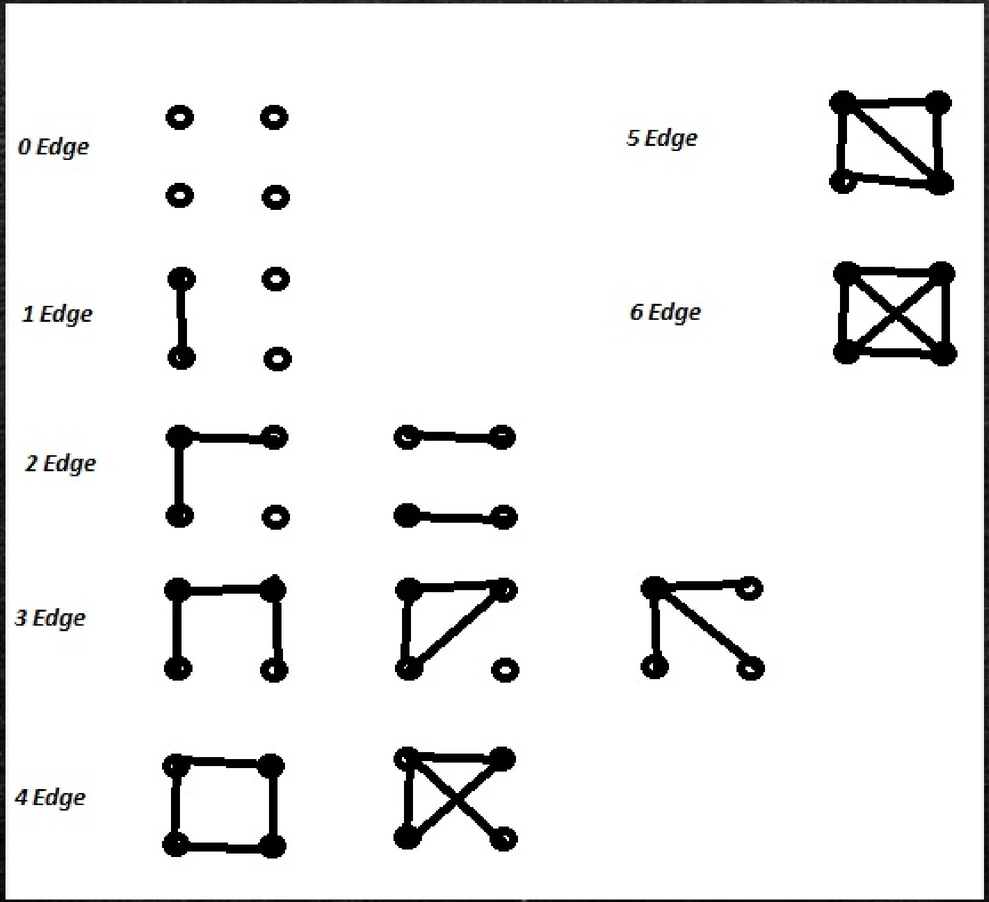
$\overline{G}$

V(G)={A,B,C,D,E}
V($\overline{G}$)=V(G)={A,B,C,D,E}

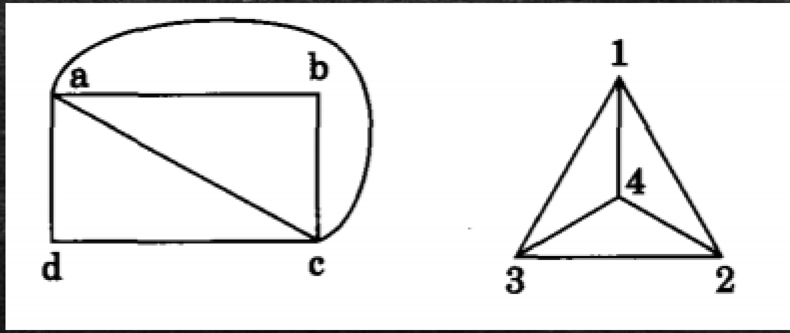
E(G)={AE,AB,BC,CD,DE}
E($\overline{G}$)={AD,AC,BE,BD,EC}

Similarly :Complement of $\overline{G}$ is G.
Hence proved.

1.c)Draw at least 3 non-isomorphic graphs on 4 vertices.[December 2016,5 marks]



1.c)Determine whether the following graphs are isomorphic. If yes, justify your answer. [December 2016,December 2010,4 marks]



Number of vertices in G= 4
Number of vertices in H=4

Number of edges in G=6
Number of edges in H=6

Degree sequence of G : {4,4,2,2}
Degree sequence of H : {3,3,3,3}

Degree sequences of graph G and H are different, therefore the two graphs are non-isomorphic.

1.d)What is an undirected graph ? Prove that an undirected graph has even number vertices of odd degree.[December 2016, 4 marks]

Undirected graph G is a finite non-empty set V together with set E containing pairs of points of V. V is called the vertex set and E is the edge set of G. In undirected graph, E(G) will be symmetric on V(G). If (u,v) is there, then (v,u) will be there.

Any graph can only have an even number of odd vertices.

Consider a (p,q) graph with {x1,x2,.....xt} is a set of odd vertices and {xt+1,.....xp} is a set of even vertices.

Let $d_G(x_i)=2c_i+1$ $1 \le i \le t$ and $d_G(x_i)=2r_i$ $t+1 \le i \le p$

Then Theorem 1 says that $2q=\sum_{1}^{p} d_G(x_i)$

$$\Rightarrow 2q=\sum_{1}^{t}(2c_i+1)+\sum_{t+1}^{p}(2r_i) = 2(c_1+c_2+\Lambda+c_t)+t+2(r_{t+1}+\Lambda+r_p),$$

which shows that t is even.

2.a)Define n-regular graph. Show for which value of n the following graphs are regular : (i) K_n (ii) Q_n [December 2016, 5 marks]

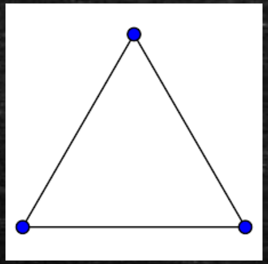
It is a graph in which each vertex has the same degree. It is said to be regular graph degree of regularity r. G is an r-regular graph where $0 \le r \le (p-1)$.

i) K_n

K_n is a regular graph with n=3.

The degree of each vertex is 2. So, K3 is regular graph.

K_n for n>3 it is (n–1)-regular.



2.c)How many edges does a complete graph of 5 vertices have ? [December 2016,2 marks]

Number of edges in a complete graph of n vertices = n(n-1)/2

In the above question,

number of vertices ,n =5

Number of edges = (n(n-1))/2

= (5(5-1))/2

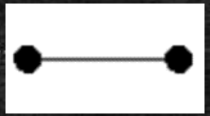
=(5*4)/2

=10

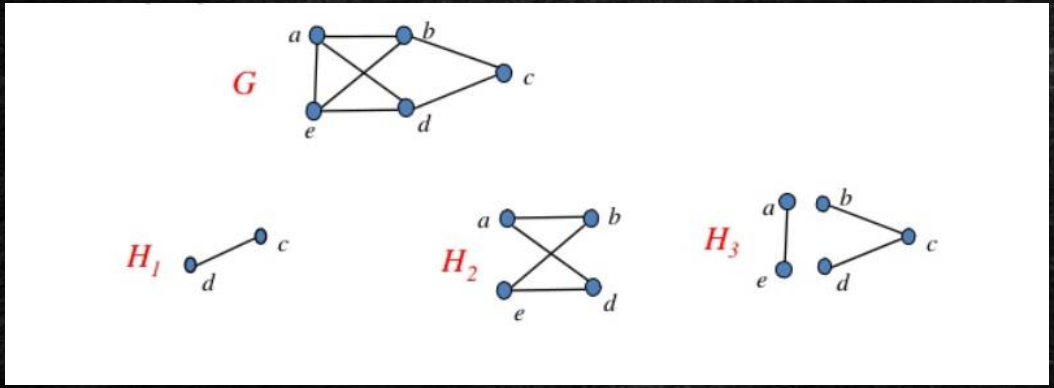
3.b)Define a graph and a subgraph. Show that for a subgraph H of a graph G Δ (H) ≤ Δ (G). [December 2016S, 5 marks]

A graph is a set of the form {(x,f(x)): x is a domain of function f}.

Example :



Let G = (V (G), E (G)) be a graph. A **subgraph** H of the graph G is a graph, such that every vertex of H is a vertex of G, and every edge of H is an edge of G also, that is, V (H) ⊆ V (G) and E (H) ⊆ E (G).



3.a)Show that for a subgraph H of a graph G Δ (H) ≤ Δ (G). [December 2014, December 2011,June 2010,December 2010, 5marks]

Let $x \in V(H)$ such that $d_H(x) = H(\Delta)$

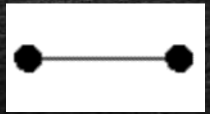
Then, $N_H(x) \subseteq N_G(x)$. Thus,

$\Delta (H)=| N_H (x)| \leq | N_G (x)|\leq\Delta (G)$

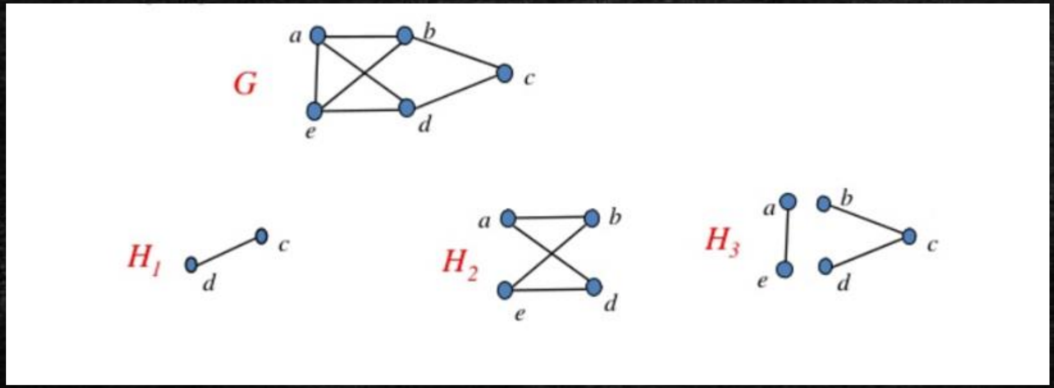
2.d)Define Graph and Subgraph. Give an example of a subgraph H of a graph G with δ (G) < δ (H) and Δ (H) ≤ Δ (G).[June 2015,4 marks]

A graph is a set of the form {(x,f(x)): x is a domain of function f}.

Example :



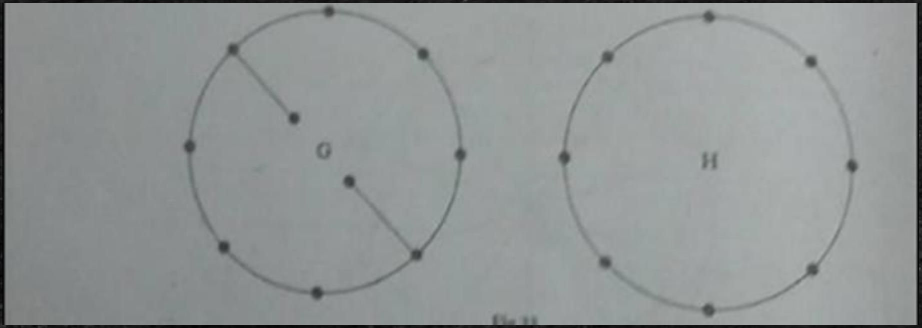
Let G = (V (G), E (G)) be a graph. A **subgraph** H of the graph G is a graph, such that every vertex of H is a vertex of G, and every edge of H is an edge of G also, that is, V (H) ⊆ V (G) and E (H) ⊆ E (G).



$\delta(G)=1<2=\delta(H)$

$\Delta(H)=2<3=\Delta(G)$

Diagram :

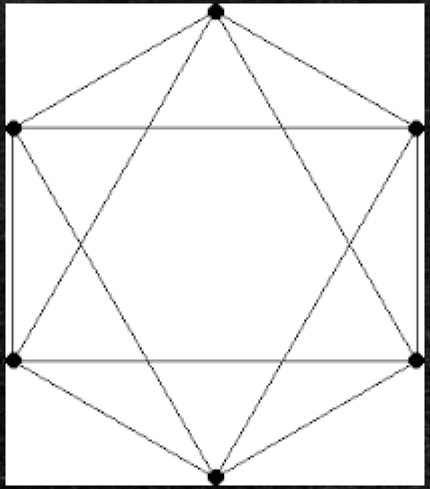


1.a)Define regular graph. Find the number of edges of a 4-regular graph with 6 vertices.[December 2015,3 marks]

It is a graph in which each vertex has the same degree. It is said to be regular graph degree of regularity r. G is an r-regular graph where $0 \leq r \leq (p-1)$.

K_n is a regular graph with degree of regularity (n-1) when n > 3.

4-regular graph with 6 vertices:



Number of edges =12

3.c)Define isomorphic graph. Give an example of the same.[December 2015, 2 marks]

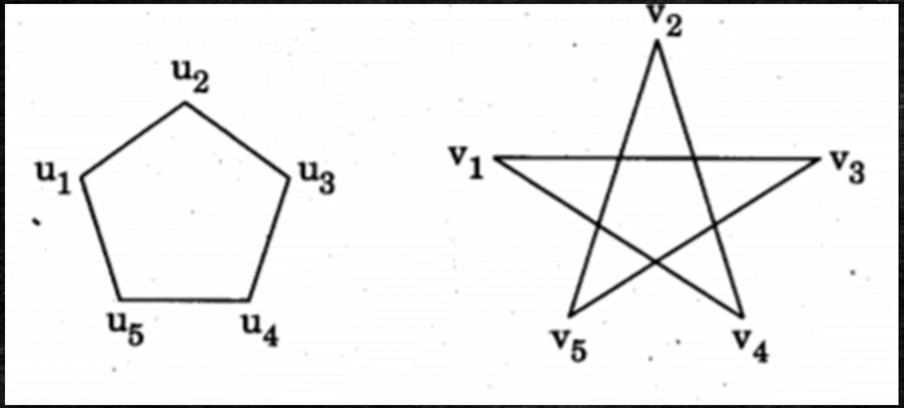
Let G=(V(G),E(G)) and H=(V(H),E(H)) be two graphs. Let us map a function f: V(G)-> V(H).

Then two graphs are said to be isomorphic, if

i) F is one-one and onto, and

ii) xy ∈ E(G) if and only if f(x) f(y) ∈ E(H)

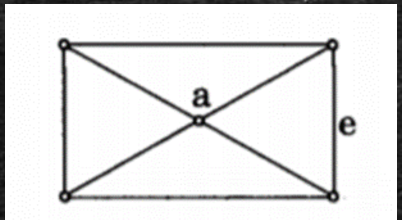
If not they are called non-isomorphic graphs.



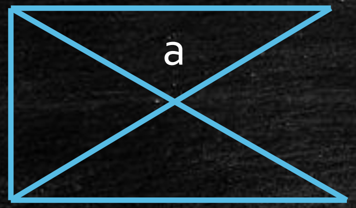
Both are (5, 5)-graphs. Degree sequence of both the graphs is <2,2,2,2,2>. Both these graphs have a copy of C5. Therefore, both these graphs are isomorphic.

4.b)For the following graph G, draw subgraphs 3 (i) G - e (ii) G - a . [December 2015,3 marks]

Graph G:



i) G-e

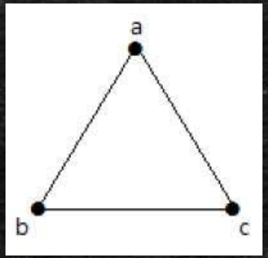


ii)

Define : (i) Simple graph (ii) Finite and infinite graph (iii) Isolated vertex (iv) Subgraph[June 2014, 4 marks]

i) Simple graph :

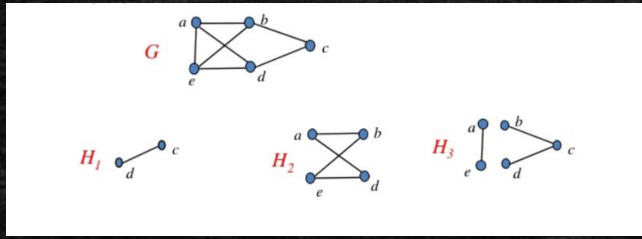
Undirected graph that has no loops or multiple edges is called a simple graph. When an edge joins a vertex to itself is called a loop. Two or more edges that joins the same vertices are parallel or multiple edges.



ii) Finite and infinite graph : A graph with a finite number of vertices and edges is called a finite graph. A graph with a finite number of nodes and edges.

iii)Isolated vertex : Vertex with degree zero is called an isolated vertex.

iv) Subgraph : Let G = (V (G), E (G)) be a graph. A **subgraph** H of the graph G is a graph, such that every vertex of H is a vertex of G, and every edge of H is an edge of G also, that is, V (H) ⊆ V (G) and E (H) ⊆ E (G).



1.f)How many edges are there in a graph with 10 vertices each of degree 6 ? [June 2014,3 marks]

According to Handshaking theorem,

$$2q = \sum_{i=1}^{p} d_i$$

q: number of edges

p: number of vertices

d_i: degree of vertex I

In the above question : p=10, d(i)=6

2q=10*6=60

q=30

3.b)Define Isomorphism of two graphs. Find whether the given graphs are isomorphic or not.[June 2014,5 marks]

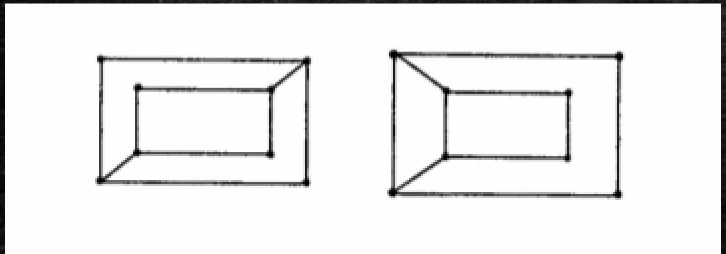
Let G=(V(G),E(G)) and H=(V(H),E(H)) be two graphs. Let us map a function f: V(G)-> V(H).

Then two graphs are said to be isomorphic, if

i) F is one-one and onto, and

ii) xy ∈ E(G) if and only if f(x) f(y) ∈ E(H)

If not they are called non-isomorphic graphs.



Number of vertices in first graph = 8

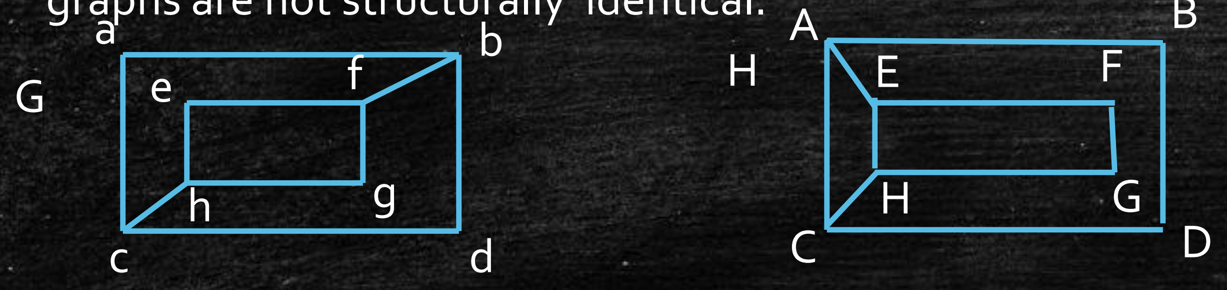
Number of vertices in second graph = 8

Number of edges in first graph = 10

Number of edges in second graph = 10

Degree sequence of both the graphs is : <3,3,3,3,2,2,2,2>

These conditions satisfies but still the graphs are non-isomorphic. This is because the two graphs are not structurally identical.



In graph G, A is a vertex of degree 2, which must corresponds to either B, D, F or G in H.

Each of these four vertices in H is adjacent to another vertex of degree two in H, which is not true for a in G.

Therefore, these are not isomorphic.

5.b)State and prove Handshaking Theorem.[June 2014, 5 marks]

Handshaking problem :

If G is a (p,q) graph with $V(G)=\{V_1....V_p\}$ and $d_i=d_G(V_i)$, $1\leq i\leq p$, then $2q = \sum_{i=1}^{p} d_i$

Proof: Consider the set $S = \{(x, e): x \in V(G), e \in E(G), x$ is an endpoint of $e\}$. Choose a vertex $v_i \in V$. This can be done in p ways. Now, since $d_i = d(v_i)$, there are precisely d_i edges incident with this vertex v_i. These edges give d_i elements of the set S. Adding over all the vertices of G, we get

$$|S| = \sum_{i=1}^{p} d_i . \qquad (1)$$

Now choose an edge e in E (G). This can be done in q ways. This edge has precisely two endpoints, and they give two elements of S. Summing over every edge $e \in E(G)$, we get

$$|S| = 2q \qquad (2)$$

This is because every edge is counted twice, once for each vertex it contains. Equating (1) and (2) we get the required result.

1.d)State and prove Handshaking Theorem.[December 2014,December 2010, 4 marks]

Handshaking problem :

If G is a (p,q) graph with $V(G)=\{V_1....V_p\}$ and $d_i=d_G(V_i)$, $1\le i\le p$, then

$$2q = \sum_{i=1}^{p} d_i$$

Proof: Consider the set $S = \{(x, e): x \in V(G), e \in E(G), x \text{ is an endpoint of } e\}$. Choose a vertex $v_i \in V$. This can be done in p ways. Now, since $d_i = d(v_i)$, there are precisely d_i edges incident with this vertex v_i. These edges give d_i elements of the set S. Adding over all the vertices of G, we get

$$|S| = \sum_{i=1}^{p} d_i . \qquad (1)$$

Now choose an edge e in E (G). This can be done in q ways. This edge has precisely two endpoints, and they give two elements of S. Summing over every edge $e \in E(G)$, we get

$$|S| = 2q \qquad (2)$$

This is because every edge is counted twice, once for each vertex it contains. Equating (1) and (2) we get the required result.

3.a)Show that for a subgraph H of a graph G Δ (H) ≤ Δ (G). [December 2014, December 2011,June 2010,December 2010, 5marks]

Let $x \in V(H)$ such that $d_H(x) = H(\Delta)$

Then, $N_H(x) \subseteq N_G(x)$. Thus,

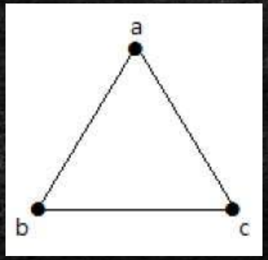
$\Delta (H)=| N_H (x)| \leq | N_G (x)|\leq\Delta (G)$

1.a)Define : 4 (i) Graph (ii) Simple Graph (iii) null graph (iv) connected Graph [December 2013,4 marks]

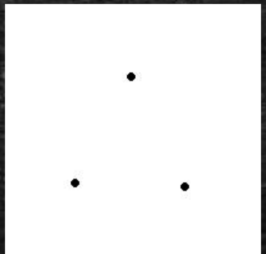
i) Graph :It is a set of the form {(x,f(x)): x is a domain of function f}.

ii) Simple graph :

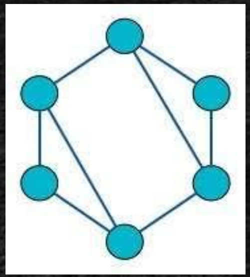
Undirected graph that has no loops or multiple edges is called a simple graph. When an edge joins a vertex to itself is called a loop. Two or more edges that joins the same vertices are parallel or multiple edges.



iii) Null graph : A graph with isolated vertices and no edges is called a null graph. It is also known as an empty graph.

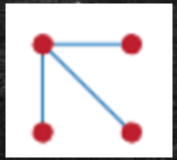


iv) Connected graph : A graph is connected when there is a path between every pair of vertices. In a connected graph, there are no unreachable vertices.



1.d)Define δ (G) and Δ (G) for a graph G.[December 2013,2 marks]

δ(G) is called the minimum vertex degree of G. It is min{$d_G(x)$:x∈ V(G)} . It is a non-negative integer.

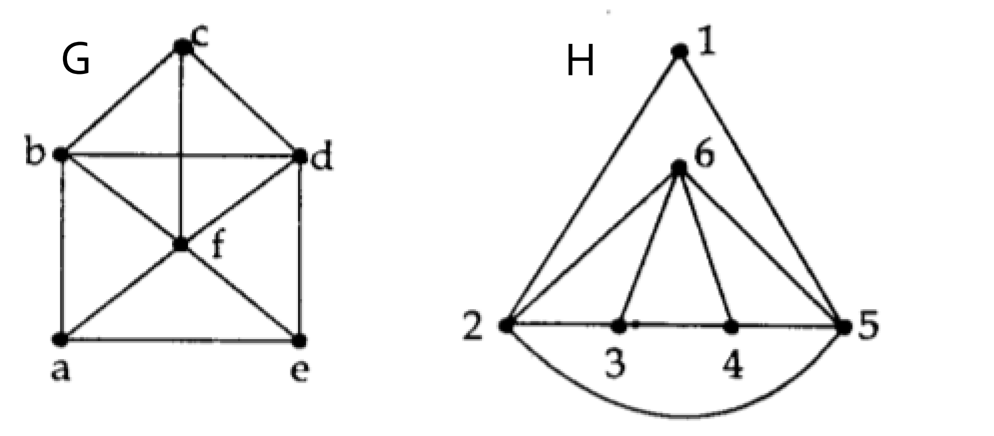


δ(G)=1

Δ (G) is called the maximum vertex degree of G. It is max{$d_G(x)$:x∈ V(G)}. It is a non-negative integer.

Δ(G)=3

4.b)Are the following graphs isomorphic ? If Yes or No justify.[December 2013,June 2010,4 marks]



Number of vertices in G = 6

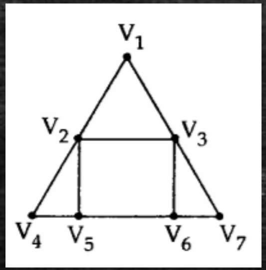
Number of vertices in H = 6

Number of edges in G = 11

Number of edges in H = 10

The two graphs have different number of edges. Therefore, the two graphs are not isomorphic.

1.d)Find the degree of each vertex in the given graph.[June 2012,4 marks]



Degree of each vertex in the above graph is :

d(v1)=2

d(v2)=4

d(v3)=4

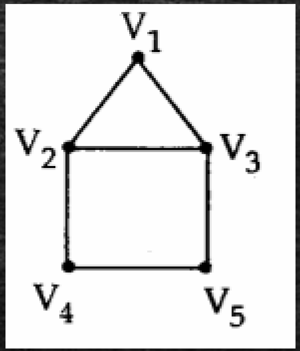
d(v4)=2

d(v5)=3

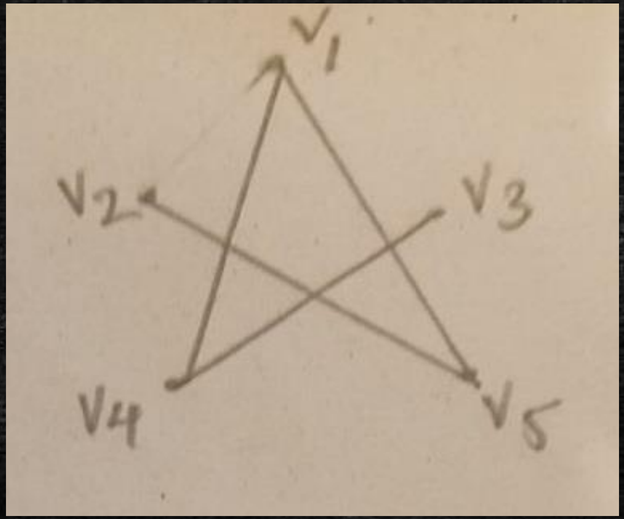
d(v6)=

d(v7)=2

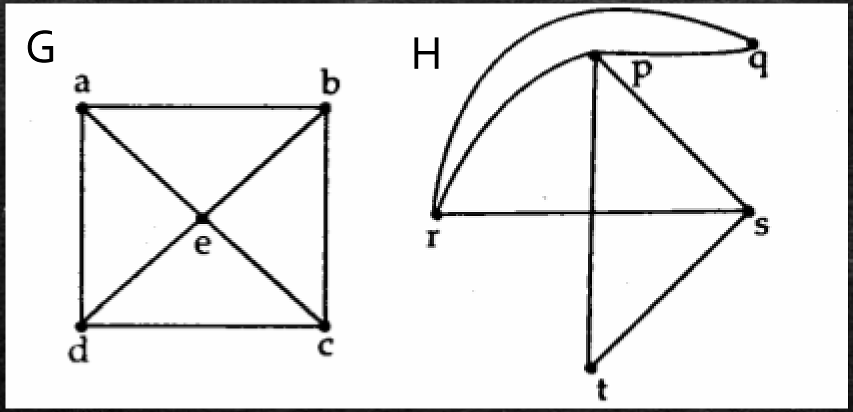
1.e)What is the complement of the given graph.[June 2012,4 marks]



Complement of the above graph :



2.a)Determine whether the graphs are isomorphic. [June 2012,5 marks]



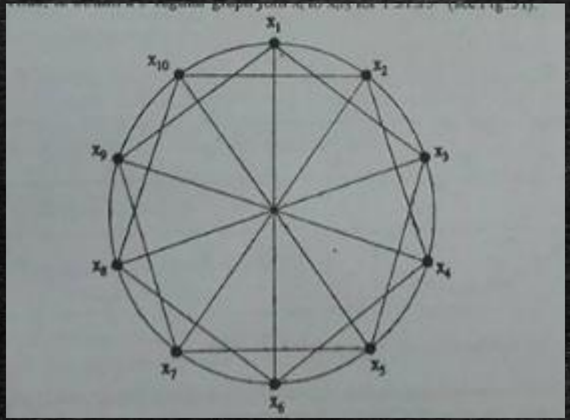
V(G)=5 V(H)=5

E(G)=8 E(H)=7

Number of edges is not the same in G and H.

Therefore, the graphs are not isomorphic.

1.b)Construct a 5-regular graph on 10 vertices.[December 2012,June 2010,3 marks]



1.b)A graph G is said to be **self complementary** if it is isomorphic to its complement G . Show that for a self complementary (p, q)-graph G, either p or (p – 1) is divisible by 4.[June 2011,4 marks]

Suppose G is a (p, q)-graph.

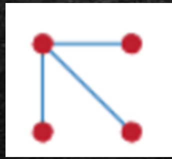
Then E (G) ∪ E($\overline{G}$) = {the set of all pairs of vertices in V (G)}.

Thus, q+$\overline{q}$ =(p(p-1))/2

If the graph G is self complementary, then q = $\overline{q}$. Thus, p (p –1) = 2q + 2q = 4q, that is 4 divides p (p –1). Since only one of p or (p –1) is even, this means either p or (p – 1) is divisible by 4.

1.c)Define minimum vertex degree of G (δ (G))and maximum vertex degree of G (Δ (G)).[June 2011,3 marks]

δ(G) is called the minimum vertex degree of G. It is min{$d_G(x)$:x∈ V(G)} . It is a non-negative integer.



δ(G)=1

Δ (G) is called the maximum vertex degree of G. It is max{$d_G(x)$:x∈ V(G)}. It is a non-negative integer.

Δ(G)=3

5.a)Can a simple graph exist with 15 vertices, with each of degree five ? Justify your answer.[June 2011,3 marks]

A corollary in graph theory states that "Any graph can only have an even number of odd vertices". This is because of the handshaking problem.

$$2q = \sum_{i=1}^{p} d_i$$

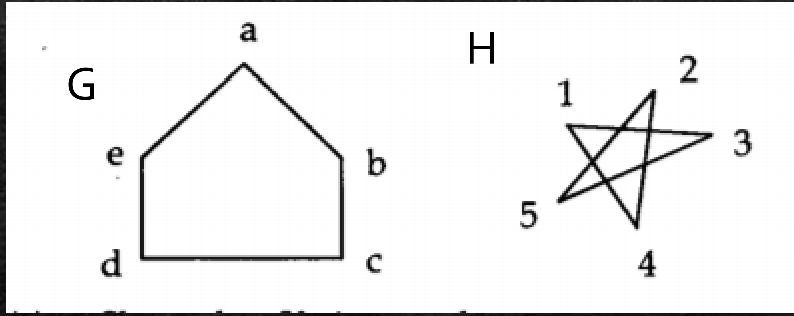
According to the question,

Number of vertices, p=15

Degree of each vertices, $D(V_i)=5$

This graph has 15 odd vertices which is odd, so the above graph cannot exist.

5.b)Are the following graphs are isomorphic ? 4 If Yes or No Justify.[June 2011,4 marks]



Number of vertices in G=5

Number of vertices in H=5

Number of edges in G=5

Number of edges in H=5

Degree sequence of G : <2,2,2,2,2>

Degree Sequence of H : <2,2,2,2,2>

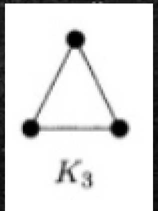
f(a)=2 f(b)=4 f(c)=1 f(d)=3 f(e)=5

This shows that the two graphs are isomorphic.

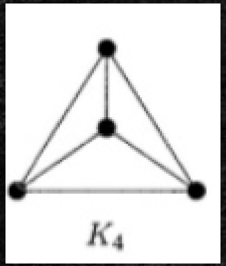
4.a)Define the concept of a complete graph. Draw complete graph each for the case when number of vertices is given by : n=3, n=4.[June 2010,3 marks]

Complete graph : Graph in which any two vertices are adjacent, i.e. each vertex is joined to every other vertex by a vertex. A complete graph on n vertices is represented by K_n.

n=3



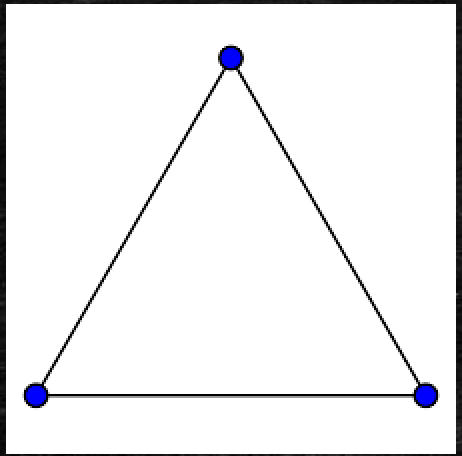
n=4



1.c)Define r-regular graph. Give an example of 3-regular graph.[December 2010,3 marks]

It is a graph in which each vertex has the same degree. It is said to be regular graph degree of regularity r. G is an r-regular graph where 0≤r≤(p-1).

K_n is a regular graph with degree of regularity (n-1) when n > 3.



1.b)Show that the sum of the degrees of all vertices of a graph is twice the number of edges in the graph.[June 2009,3 marks]

Sum of the degrees of al vertices of a graph is twice the number of edges in the graph. This is called handshaking problem.

Proof: Consider the set S = {(x, e): x $\in$ V(G), e $\in$ E(G), x is an endpoint of e}. Choose a vertex $v_i \in V$. This can be done in p ways. Now, since $d_i = d(v_i)$, there are precisely d_i edges incident with this vertex v_i. These edges give d_i elements of the set S. Adding over all the vertices of G, we get

$$|S| = \sum_{i=1}^{p} d_i \,. \qquad (1)$$

Now choose an edge e in E (G). This can be done in q ways. This edge has precisely two endpoints, and they give two elements of S. Summing over every edge $e \in E(G)$, we get

$$|S| = 2q \qquad (2)$$

This is because every edge is counted twice, once for each vertex it contains. Equating (1) and (2) we get the required result.

1.c)Define isomorphism of graphs. Determine whether the graphs are isomorphic.

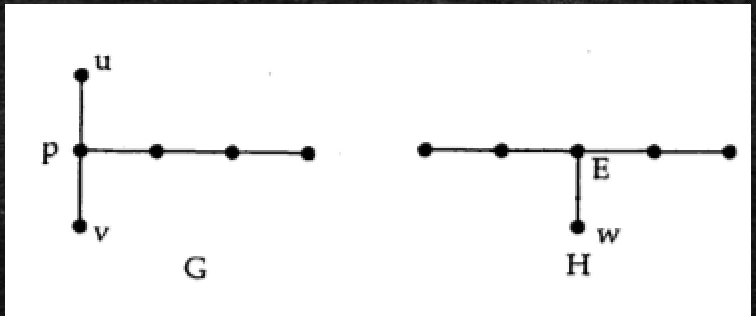
Let G=(V(G),E(G)) and H=(V(H),E(H)) be two graphs. Let us map a function f: V(G)-> V(H).

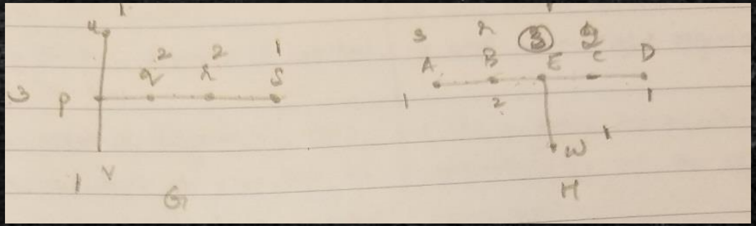
Then two graphs are said to be isomorphic, if

i) F is one-one and onto, and

ii) xy ∈ E(G) if and only if f(x) f(y) ∈ E(H)

If not they are called non-isomorphic graphs.





Number of vertices in G=6

Number of vertices in H=6

Number of edges in G=5

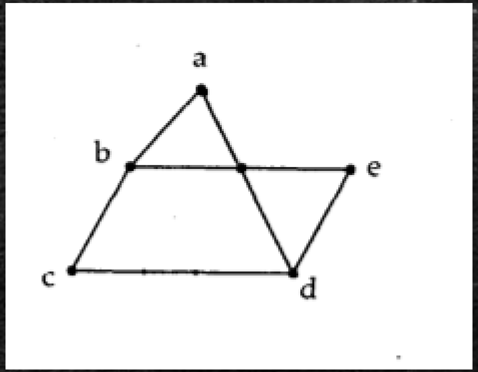
Number of edges in H=5

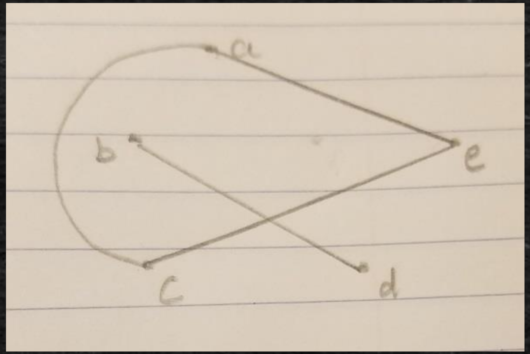
Degree sequence of G : <3,2,2,1,1,1>

Degree sequence of H : <3,2,2,1,1,1>

The two graphs are not isomorphic. This is because in graph G, vertex p with degree 3 is adjacent to two vetices of degree 1 (u,v) and a vertex with degree 2 (q). This is not the case in graph H(vertex with degree 3 is adjacent to two vertices with degree 2 and a vertex with degree 1).

1.f) What is the complement of the given graph?[June 2009,3 marks]





3.b)How many vertices will the following graphs have if they contain :[June 2009,4 marks]

i) 16 edges and all vertices of degree 2.

Sum of all degrees of vertices=2* number of edges

Let number of vertices be n.

2 * n= 2*16

n=16

ii) 21 edges, 3 vertices of degree 4 and the other vertices of degree 3

Let n be the number of vertices.

(3*4) +(n*3)=2*21

12+3n=42

$30=3n$

$n=10$

1.b)The number of vertices of odd degree in a graph is always even.[December 2009,3 marks]

Any graph can only have an even number of odd vertices.

Consider a (p,q) graph with {x1,x2,.....xt} is a set of odd vertices and {xt+1,.....xp} is a set of even vertices.

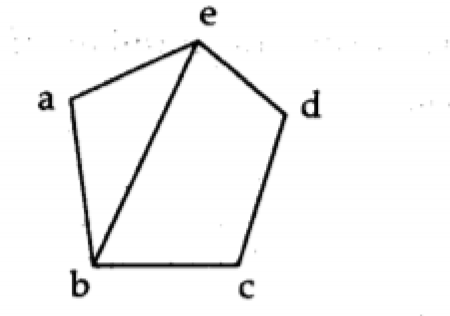
Let $d_G(x_i)=2c_i+1 \quad 1\le i\le t$ and $d_G(x_i)=2r_i \quad t+1\le i\le p$

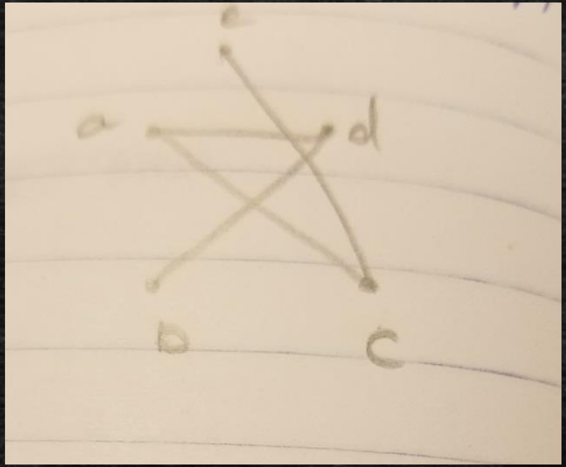
Then Theorem 1 says that $2q=\sum_{1}^{p} d_G(x_i)$

$$\Rightarrow 2q=\sum_{1}^{t}(2c_i+1)+\sum_{t+1}^{p}(2r_i) = 2(c_1+c_2+\Lambda+c_t)+t+2(r_{t+1}+\Lambda+r_p),$$

which shows that t is even.

1.c)What is the complement of the given graph?[December 2009,2 marks]



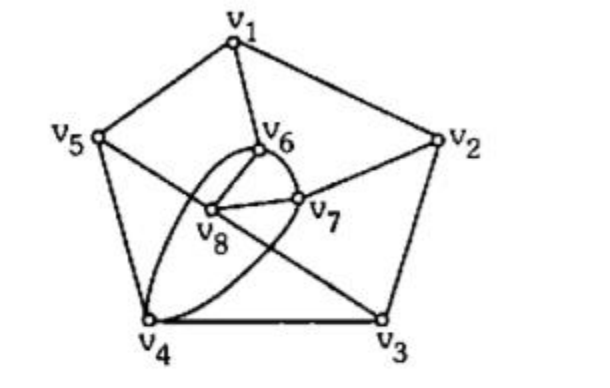


4.b)What is the largest number of vertices in a graph with 35 edges if all vertices are of degree at least 3 ?[December 2009,5 marks]

Maximum degree of a graph >= Sum of degree of individual vertices
2E >= deg(V1) + deg(V2) + … + deg(Vn)
2 * 35 >= 3 + 3 + … + 3 …(I),
70 >= 3n
23.33 >= n or 23 >= n

1.a)Consider the graph below :[June 2008,2 mark]

i) Find δ (G) and , Δ (G)

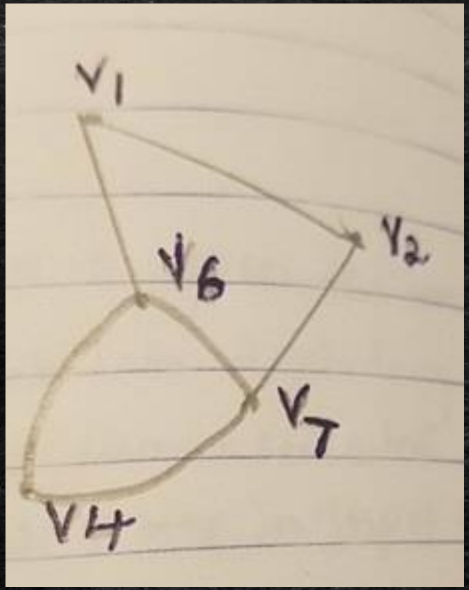


Degree of v1 = 3 Degree of v2 = 3 Degree of v3 = 3 Degree of v4 = 4 Degree of v5 = 3

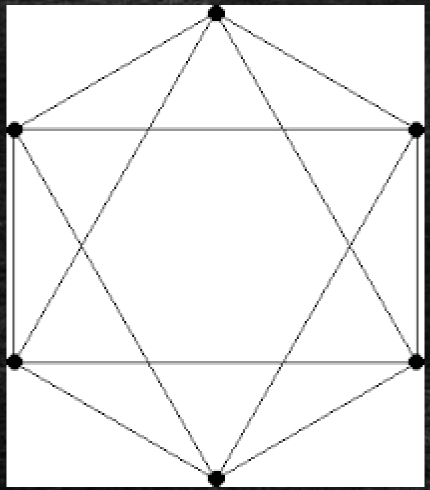
Degree of v6 = 4 Degree of v7 = 4 Degree of v8 = 4

From the above diagram, δ (G) = 3 Δ (G) = 4

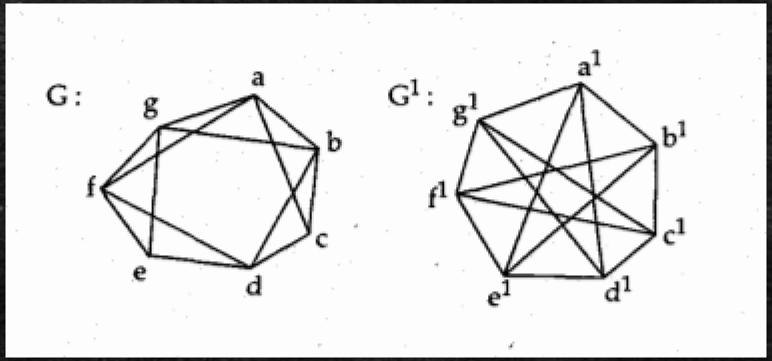
ii) Draw the subgraph induced by the set {v1,v6, v4, v7,v2}



3.c)Draw a 4-regular graph on 6 vertices.[June 2008,2 marks]



1.a)Show that the graphs G and G’ are isomorphic.[December 2008,4 marks]



Number of vertices in G=7

Number of edges= 13

Number of vertices in G′= 7

Number of edges= 14

Therefore both the graphs are not isomorphic.